Finite state transducers

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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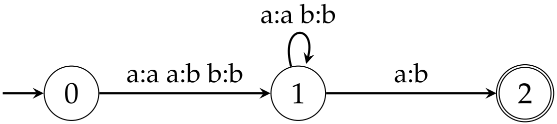
Finite state transducers

A quick introduction

- A *finite state transducer* (FST) is a finite state machine where transitions are conditioned on pairs of symbols
- The machine moves between the states based on an *input* symbol, while it outputs the corresponding *output* symbol
- An FST encodes a *relation*, a mapping from a set to another
- The relation defined by an FST is called a *regular* (or *rational*) relation



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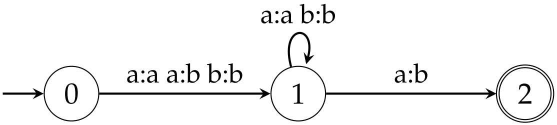


babba $\longrightarrow$ babbb

aba $\longrightarrow$ bbb

aba $\longrightarrow$ abb

Formal definition

A finite state transducer is a tuple $(\Sigma_i, \Sigma_o, Q, q_0, F, \Delta)$

- Σ_i is the *input* alphabet
- Σ_o is the *output* alphabet
- Q a finite set of states
- q_0 is the start state, $q_0 \in Q$
- F is the set of accepting states, $F \subseteq Q$
- Δ is a relation $(\Delta : Q \times \Sigma_i \to Q \times \Sigma_o)$

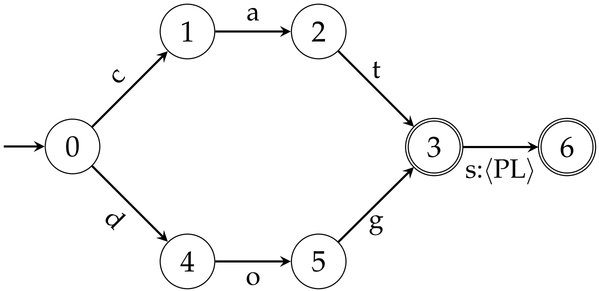
Where do we use FSTs?

Uses in NLP/CL

- Morphological analysis
- Spelling correction
- Transliteration
- Speech recognition
- Grapheme-to-phoneme mapping
- Normalization
- Tokenization
- POS tagging (not typical, but done)
- partial parsing / chunking
- ...

Where do we use FSTs?

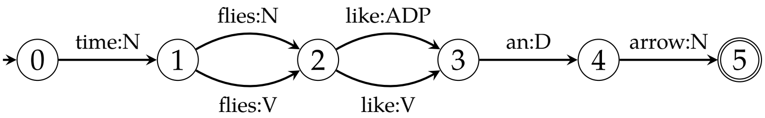
example 1: morphological analysis

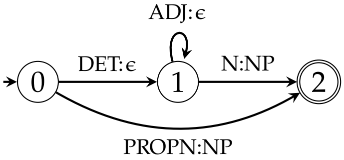


In this lecture, we treat an FSA as a simple FST that outputs its input: the edge label ‘a’ is a shorthand for ‘a:a’.

Where do we use FSTs?

example 2: POS tagging / shallow parsing





Note: (1) It is important to express the ambiguity. (2) This gets interesting if we can ‘compose’ these automata.

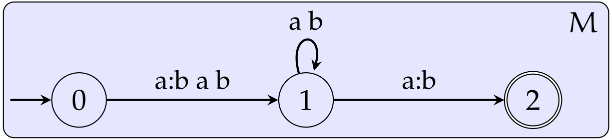
Closure properties of FSTs

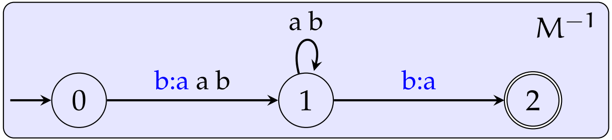
Like FSA, FSTs are closed under some operations.

- Concatenation
- Kleene star
- ~~Complement~~
- Reversal
- Union
- ~~Intersection~~
- *Inversion*
- *Composition*

FST inversion

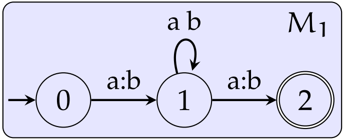
- Since an FST encodes a relation, it can be reversed
- Inverse of an FST swaps the input symbols with output symbols
- We indicate inverse of an FST M with M^{-1}

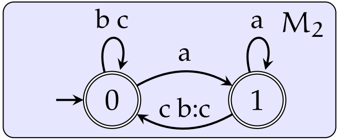




FST composition

sequential application

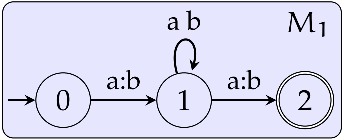


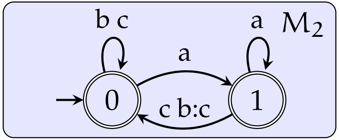


$$aa \xrightarrow{M_1}$$
$$bb \xrightarrow{M_1}$$
$$aaaa \xrightarrow{M_1}$$
$$abaa \xrightarrow{M_1}$$

FST composition

sequential application





$$\mathtt{aa} \xrightarrow{M_1} \mathtt{bb} \xrightarrow{M_2}$$

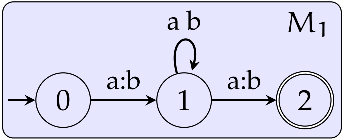
$$\mathtt{bb} \xrightarrow{M_1} \varnothing \xrightarrow{M_2}$$

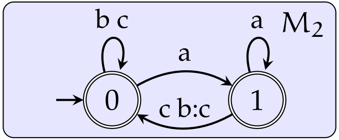
$$\mathtt{aaaa} \xrightarrow{M_1} \mathtt{baab} \xrightarrow{M_2}$$

$$\mathtt{abaa} \xrightarrow{M_1} \mathtt{bbab} \xrightarrow{M_2}$$

FST composition

sequential application



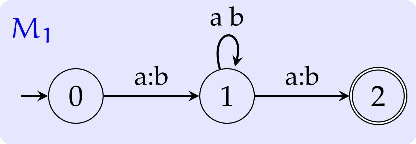


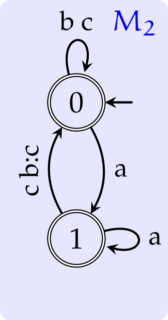
$$\xrightarrow{\qquad\qquad M_1 \circ M_2 \qquad\qquad}$$

$$
\begin{array}{lllll}
aa & \xrightarrow{M_1} & bb & \xrightarrow{M_2} & bb \\
bb & \xrightarrow{M_1} & \varnothing & \xrightarrow{M_2} & \varnothing \\
aaaa & \xrightarrow{M_1} & baab & \xrightarrow{M_2} & baac \\
abaa & \xrightarrow{M_1} & bbab & \xrightarrow{M_2} & bbac
\end{array}
$$

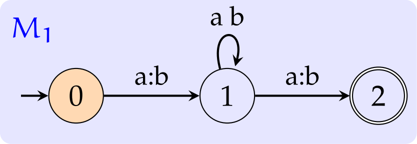
- Can we compose two FSTs without running them sequentially?

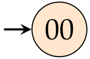
FST composition

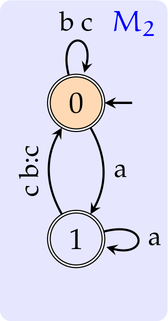




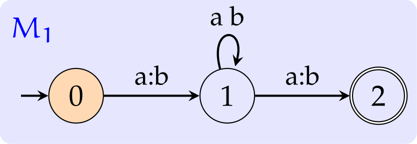
FST composition

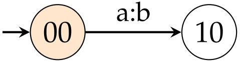


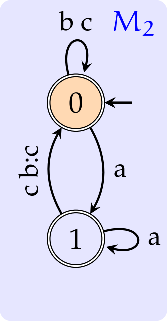




FST composition







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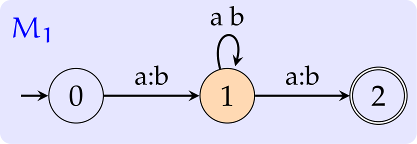

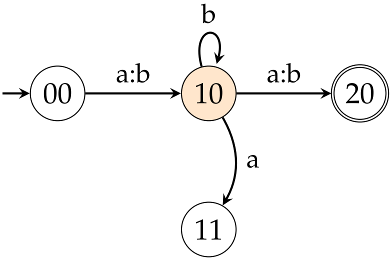

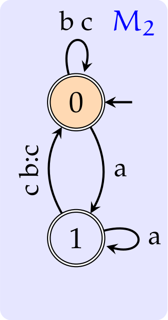

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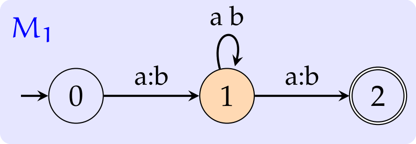

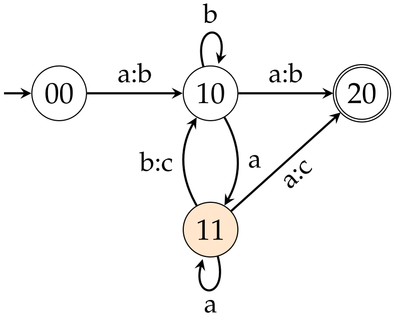

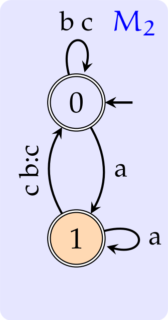

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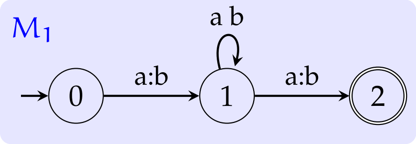
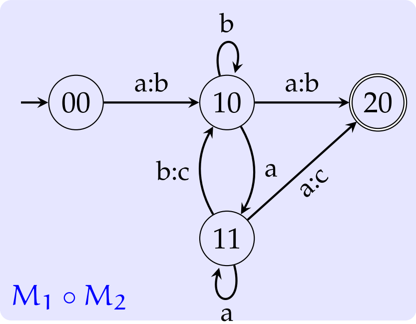
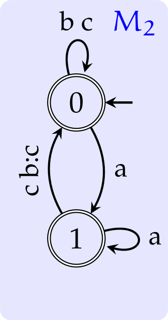
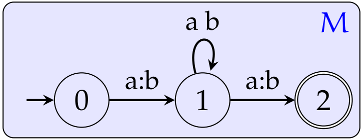


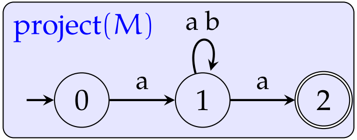




Projection

- *Projection* turns an FST into a FSA, accepting either the input language or the output language





FST determinization

- A deterministic FST has unambiguous transitions from every state on any *input* symbol
- We can extend the subset construction to FSTs
- Determinization of FSTs means converting to a *subsequential* FST
- However, not all FSTs can be determinized

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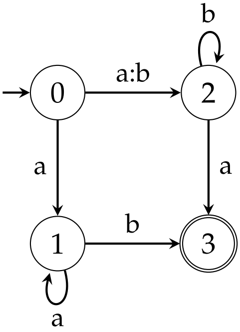
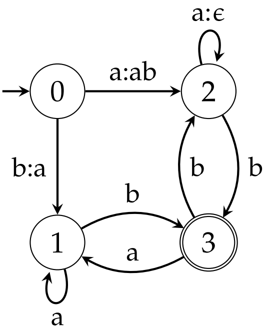
Is this FST deterministic?



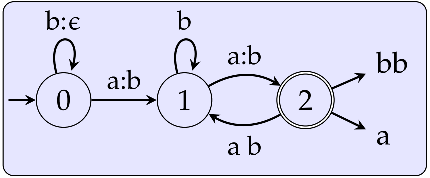
Sequential FSTs

- A sequential FST has a single transition from each state on every *input* symbol
- Output symbols can be strings, as well as ϵ
- The recognition is linear in the length of input
- However, sequential FSTs do not allow ambiguity



Subsequential FSTs

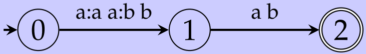
- A *k-subsequential* FST is a sequential FST which can output up to k strings at an accepting state
- Subsequential transducers allow limited ambiguity
- Recognition time is still linear



- The 2-subsequential FST above maps every string it accepts to two strings, e.g.,
  - baa → bba
  - baa → bbbb

An exercise

Convert the following FST to a subsequential FST



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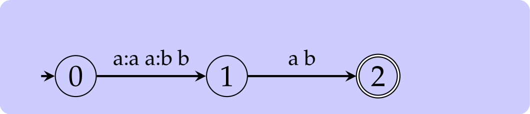

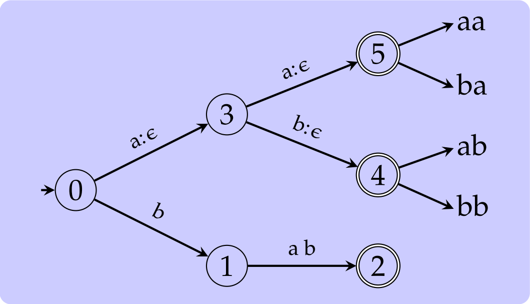

Determinizing FSTs

Another example

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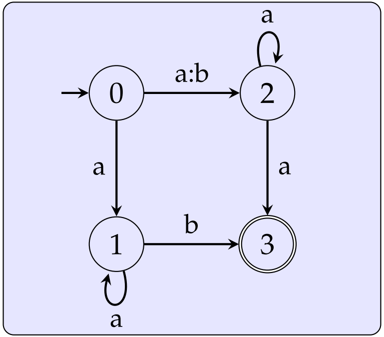

Determinizing FSTs

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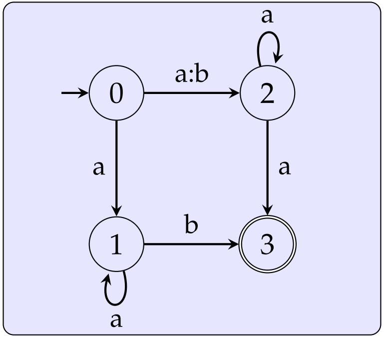

Note that we cannot ‘determine’ the output on first input until reaching the final input.

FSA vs FST

- FSA are *acceptors*, FSTs are *transducers*
- FSA accept or reject their input, FSTs produce output(s) for the inputs they accept
- FSA define sets, FSTs define relations between sets
- FSTs share many properties of FSAs. However,
  - FSTs are not closed under intersection and complement
  - We can compose (and invert) the FSTs
  - Determinizing FSTs is not always possible
- Both FSA and FSTs can be weighted (not covered in this course)

Next:

- FSA and regular languages
- Parsing

References / additional reading material

- Jurafsky and Martin (2009, Ch. 3)
- Additional references include:
  - Roche and Schabes (1996) and Roche and Schabes (1997): FSTs and their use in NLP
  - Mohri (2009): weighted FSTs

References / additional reading material (cont.)

- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.
- Mohri, Mehryar (2009). "Weighted automata algorithms". In: *Handbook of Weighted Automata*. Monographs in Theoretical Computer Science. Springer, pp. 213–254.
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