

NFA determinization

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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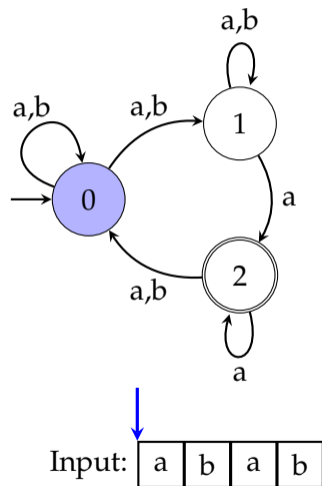
University of Tübingen
Seminar für Sprachwissenschaft

Winter Semester 2023/24

Recap

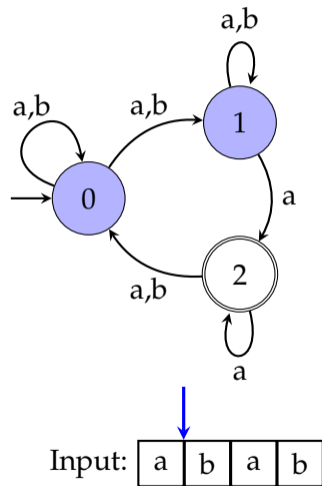
- Finite state automata come in two flavors
 - Deterministic (DFA): linear recognition time
 - Deterministic (NFA): sometimes more intuitive, easy to define, but exponential time (worst case) recognition
- The DFA and NFA are equivalent: for any language recognized by an NFA there is also a DFA recognizing the same language
- Then, the question is: how can we *determinize* an NFA to obtain an equivalent DFA

NFA recognition (again)



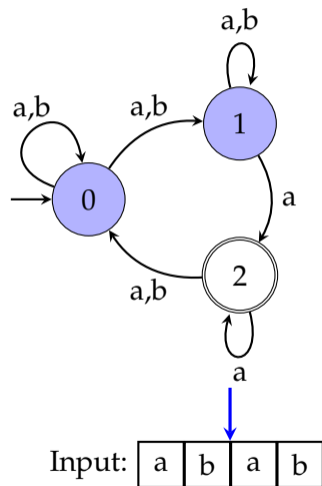
1. Start at q_0
2. Take the next input, mark all possible next states
3. If an accepting state is marked at the end of the input, accept

NFA recognition (again)



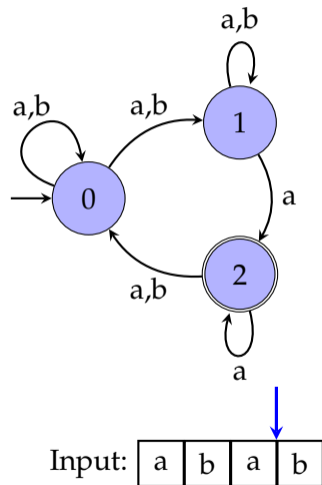
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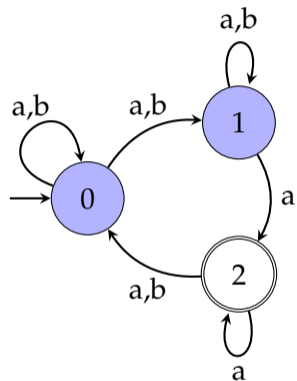
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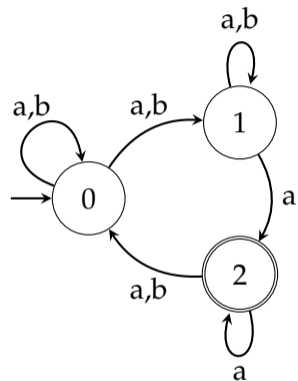


Input:

| | | | |
|---|---|---|---|
| a | b | a | b |
|---|---|---|---|

1. Start at q_0
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NFA recognition (again)



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The process is *deterministic*, and *finite-state*.

Input:

| | | | |
|---|---|---|---|
| a | b | a | b |
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Determinization

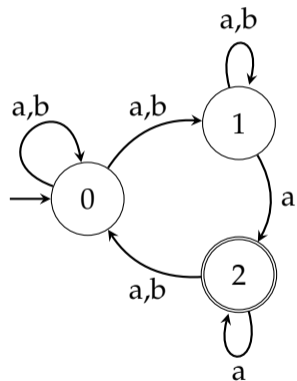
the subset construction

Intuition: remember the parallel NFA recognition. We can consider an NFA being a deterministic machine which is at a **set of states** at any given time.

- *Subset construction* (sometimes called power set construction) uses this intuition to convert an NFA to a DFA
- The algorithm can be modified to handle ϵ -transitions (or we can eliminate ϵ 's as a preprocessing step)

The subset construction

by example

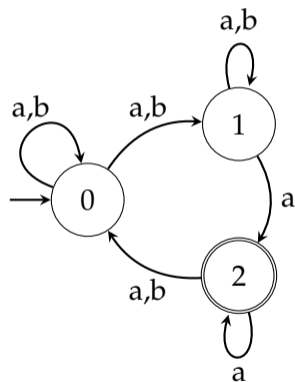


transition table with subsets

| | <i>symbol</i> | |
|---------------------|---------------|-------------|
| | a | b |
| \emptyset | \emptyset | \emptyset |
| $\rightarrow \{0\}$ | $\{0, 1\}$ | $\{0, 1\}$ |
| $\{1\}$ | $\{1, 2\}$ | $\{1\}$ |
| $* \{2\}$ | $\{0, 2\}$ | $\{0\}$ |
| $\{0, 1\}$ | $\{0, 1, 2\}$ | $\{0, 1\}$ |
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The subset construction

by example



transition table with subsets

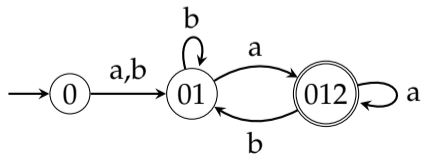
| | <i>symbol</i> | |
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The subset construction

by example: the resulting DFA

transition table without useless/inaccessible states

| | <i>symbol</i> | |
|---------------------|---------------|------------|
| | a | b |
| $\rightarrow \{0\}$ | $\{0, 1\}$ | $\{0, 1\}$ |
| $\{0, 1\}$ | $\{0, 1, 2\}$ | $\{0, 1\}$ |
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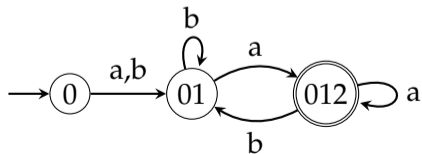


The subset construction

by example: the resulting DFA

transition table without useless/inaccessible states

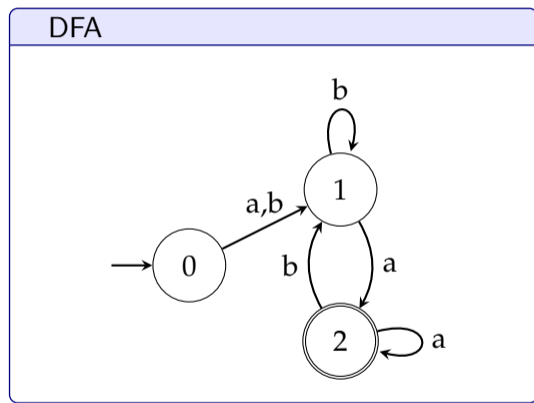
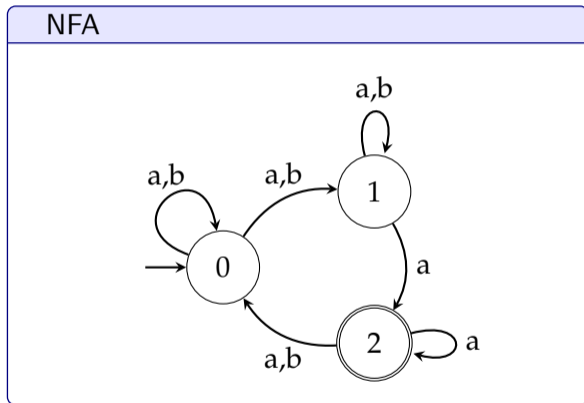
| | <i>symbol</i> | |
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Do you remember the set of states marked during parallel NFA recognition?

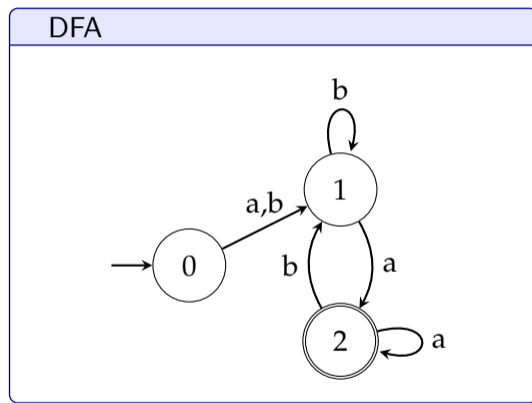
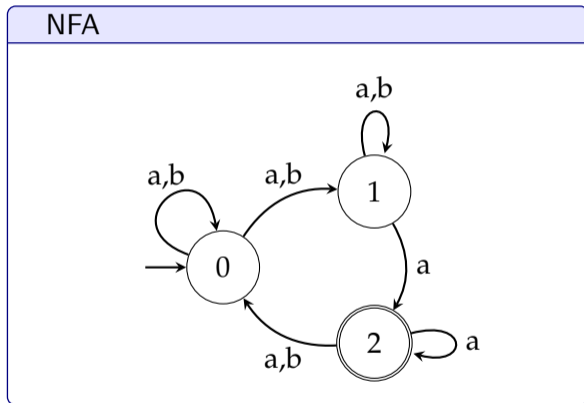
The subset construction

by example: side by side



The subset construction

by example: side by side

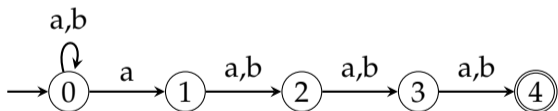


- What language do they recognize?

The subset construction

wrapping up

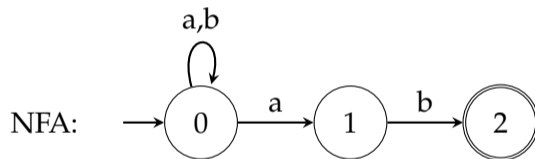
- In worst case, resulting DFA has 2^n nodes
- Worst case is rather rare, number of nodes in an NFA and the converted DFA are often similar
- In practice, we do not need to enumerate all 2^n subsets
- We've already seen a typical problematic case:



- We can also skip the unreachable states during subset construction

Yet another exercise

Determinize the following automaton





Summary

- FSA are efficient tools with many applications
- FSA have two flavors: DFA, NFA (or maybe three: ϵ -NFA)
- DFA recognition is linear, recognition with NFA may require exponential time
- Reading suggestion: Hopcroft and Ullman (1979, Ch. 2&3), Jurafsky and Martin (2009, Ch. 2)

Next:

- Minimization

Acknowledgments, credits, references

-  Hopcroft, John E. and Jeffrey D. Ullman (1979). *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley Series in Computer Science and Information Processing. Addison-Wesley. ISBN: 9780201029888.
-  Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.

