

# Top-down Chart Parsing: the Earley algorithm

## Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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# Parsing so far

- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Another aspect of a parser is its directionality. Two choices are:
  - Directional: parses processes the input left to right (right to left is also possible, but rarely used)
  - Non-directional: order is not important, typically require all input to be in memory before processing

# Top-down parsing as search

the cat bites a dog

S → NP VP  
NP → Det N  
VP → V NP  
VP → V  
Det → a  
Det → the  
N → cat  
N → dog  
V → bites

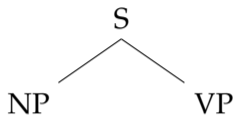
# Top-down parsing as search

S

the cat bites a dog

S	→	NP VP
NP	→	Det N
VP	→	V NP
VP	→	V
Det	→	a
Det	→	the
N	→	cat
N	→	dog
V	→	bites

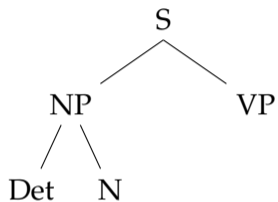
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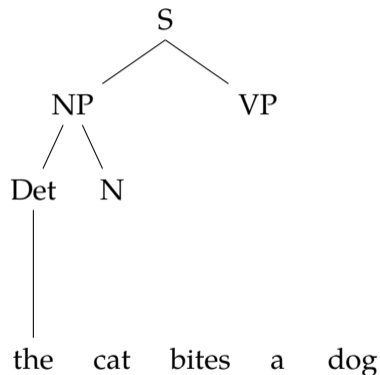
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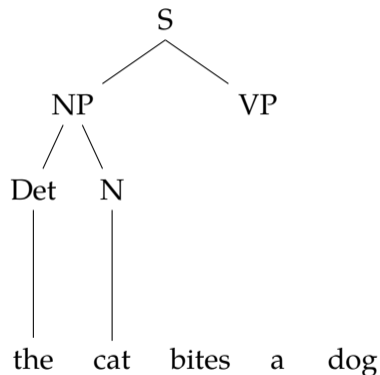
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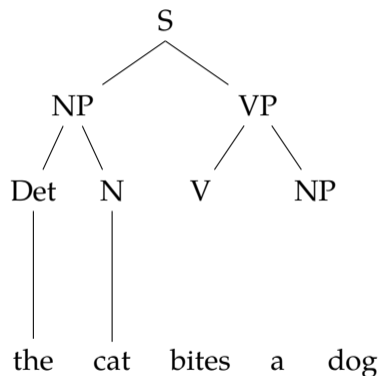
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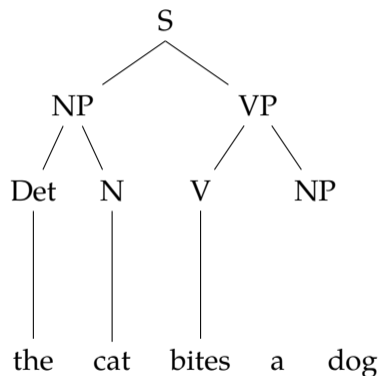


# Top-down parsing as search



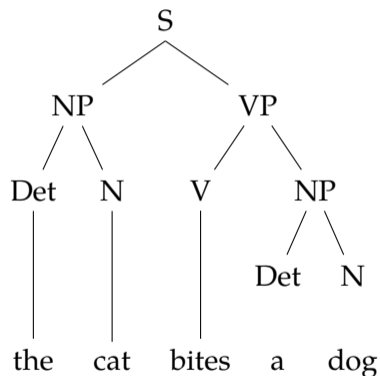
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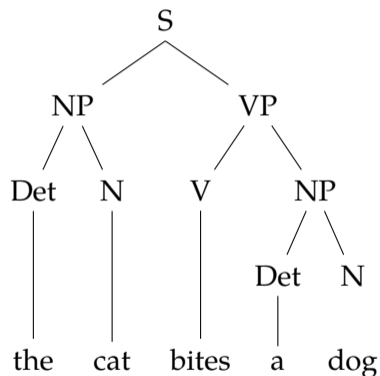
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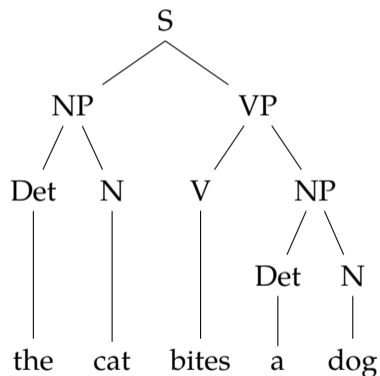
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# Top-down parsing as search



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# Earley algorithm

- Earley algorithm is a top down (and left-to-right) parsing algorithm
- It allows arbitrary CFGs
- Keeps record of constituents that are
  - predicted using the grammar (top-down)
  - in-progress with partial evidence
  - completed based on input seen so farat every position in the input string
- Time complexity is  $O(n^3)$

## Earley chart entries (states or items)

Earley chart entries are CF rules with a 'dot' on the RHS representing the state of the rule

- $A \rightarrow \bullet\alpha[i, i]$  predicted without any evidence (yet)
- $A \rightarrow \alpha \bullet \beta[i, j]$  partially matched
- $A \rightarrow \alpha\beta \bullet [i, j]$  completed, the non-terminal  $A$  is found in the given span

# Earley algorithm: an informal sketch

1. Start at position 0, predict S
2. Predict all possible states (rules that apply)
3. Read a word
4. Update the table, advance the dot if possible
5. Go to step 2
6. If we have a completed S production at the end of the input, the input is recognized



# Earley algorithm: three operations

**Predictor** adds all rules that are possible at the given state

**Completer** adds states from the earlier chart entries that match the completed state to the chart entry being processed, and advances their dot

**Scanner** adds a completed state to the next chart entry if the current category is a pre-terminal symbol, and the terminal symbol (word) matches

## Earley parsing example (chart[0])

0	she	1	saw	2	a	3	duck	4
state	rule			position		operation		
0	$\gamma \rightarrow \bullet S$			[0,0]		initialization		
1	$S \rightarrow \bullet NP VP$			[0,0]		predictor		
2	$S \rightarrow \bullet Aux NP VP$			[0,0]		predictor		
3	$NP \rightarrow \bullet Det N$			[0,0]		predictor		
4	$NP \rightarrow \bullet NP PP$			[0,0]		predictor		
5	$NP \rightarrow \bullet Prn$			[0,0]		predictor		

Note: the chart[0] is independent of the input.

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$   
 $Aux \rightarrow does | has$

## Earley parsing example (chart[1])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
6	Prn → she •	[0,1]	scanner					
7	NP → Prn •	[0,1]	completer					
8	S → NP •VP	[0,1]	completer					
9	NP → NP •PP	[0,1]	completer					
10	VP → •V NP	[1,1]	predictor					
11	VP → •V	[1,1]	predictor					
12	VP → •VP PP	[1,1]	predictor					
13	PP → •Prp NP	[1,1]	predictor					

S → NP VP  
 S → Aux NP VP  
 NP → Det N  
 NP → Prn  
 NP → NP PP  
 VP → V NP  
 VP → V  
 VP → VP PP  
 PP → Prp NP  
 N → duck  
 N → park  
 V → duck  
 V → ducks  
 V → saw  
 Prn → she | her  
 Prp → in | with  
 Det → a | the  
 Aux → does | has

## Earley parsing example (chart[2])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
14	V → saw •	[1,2]	scanner					
15	VP → V •NP	[1,2]	completer					
16	VP → V •	[1,2]	completer					
17	S → NP VP •	[0,2]	completer					
18	NP → •Det N	[2,2]	predictor					
19	NP → •NP PP	[2,2]	predictor					
20	NP → •Prn	[2,2]	predictor					

S → NP VP  
 S → Aux NP VP  
 NP → Det N  
 NP → Prn  
 NP → NP PP  
 VP → V NP  
 VP → V  
 VP → VP PP  
 PP → Prp NP  
 N → duck  
 N → park  
 V → duck  
 V → ducks  
 V → saw  
 Prn → she | her  
 Prp → in | with  
 Det → a | the  
 Aux → does | has

## Earley parsing example (chart[3])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
21	Det → a •	[2,3]	scanner					
22	NP → Det •N	[2,3]	completer					

S → NP VP  
 S → Aux NP VP  
 NP → Det N  
 NP → Prn  
 NP → NP PP  
 VP → V NP  
 VP → V  
 VP → VP PP  
 PP → Prp NP  
 N → duck  
 N → park  
 V → duck  
 V → ducks  
 V → saw  
 Prn → she | her  
 Prp → in | with  
 Det → a | the  
 Aux → does | has

## Earley parsing example (chart[4])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
23	N → duck •	[3,4]	scanner					
24	V → duck •	[3,4]	scanner					
25	NP → Det N •	[2,4]	completer					
26	VP → V NP •	[1,4]	completer					
27	S → NP VP •	[0,4]	completer					

S → NP VP  
 S → Aux NP VP  
 NP → Det N  
 NP → Prn  
 NP → NP PP  
 VP → V NP  
 VP → V  
 VP → VP PP  
 PP → Prp NP  
 N → duck  
 N → park  
 V → duck  
 V → ducks  
 V → saw  
 Prn → she | her  
 Prp → in | with  
 Det → a | the  
 Aux → does | has

# Earley parsing: summary

- Complexity (asymptotic) is the same as CKY
  - time complexity :  $O(n^3)$
  - space complexity:  $O(n^2)$
- Our example shows recognition, we need to maintain back links for parsing
- Again, the Earley chart stores a parse forest compactly, but extracting all trees may require exponential time

# Summary

- The Earley parser is a top-down parser with bottom-up filtering (or, you can also view it the other way around)
- The parser improves over a backtracking parser by
  - dynamic programming: not re-computing the subtrees
  - filtering: not generating hypotheses (predictor) that cannot match at a given input position
- It can process any CFG (no need for CNF)
- There is a nice relation between CKY and Earley: you can view Earley as binarizing the grammar (converting to CNF) 'on the fly'



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Next:

- Dependency parsing
- Reading suggestion: Jurafsky and Martin (2009, draft chapter 14)

# An exercise

Construct the CKY and Earley charts for the sentence below

The duck she saw is in the park

Recommended grammar:

S → NP VP	PP → Prp NP
NP → Det N	N → park
NP → Prn	N → duck
NP → NP PP	V → is
NP → NP S	V → saw
VP → V NP	Prn → she
VP → V	Prp → in
VP → VP PP	Det → the

# Acknowledgments, references, additional reading material



Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.